Exam III
Section I
Part A — No Calculators

1. B p. 49

A. $f'(x) = 3x^2$. Then $f'(0) = 0$, producing a horizontal tangent.
B. $f'(x) = \frac{1}{3}x^{-2/3}$. Then $f'(0)$ is undefined and $\lim_{x \to 0} f'(x) = \infty$.
C. f itself is undefined at $x = 0$. There is no point on the curve there.
D. $f'(x) = \cos x$. Then $f'(0) = 1$, producing a non-vertical tangent.
E. $f'(x) = \sec^2 x$. Then $f'(0) = 0$, producing a horizontal tangent.

The only vertical tangent is for function (B).

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2. C p. 49

$$\int_0^5 \frac{dx}{\sqrt{1+3x}} = \frac{1}{3} \cdot \int_0^5 \frac{3dx}{\sqrt{1+3x}} = \frac{2}{3}\sqrt{1+3x}\Big]_0^5 = \frac{2}{3}(4-1) = 2$$

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3. E p. 50

(A), (B), (C), and (D) are defined everywhere and have no discontinuities.
(E) is undefined at $x = -1$, and hence is discontinuous at $x = -1$.

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4. E p. 50

$$\int_0^2 e^{-x}\,dx = -e^{-x}\Big]_0^2 = -\frac{1}{e^2} + 1 = 1 - \frac{1}{e^2}$$

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5. C p. 50

$g(x) = x + \cos x$
By definition, $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x)$
Hence the value of the limit is $g'(x) = 1 - \sin x$.

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6. C p. 51

$$\int_0^4 \frac{2x}{x^2+9}\,dx = \ln\left|x^2+9\right|\Big]_0^4 = \ln 25 - \ln 9 = \ln\left[\frac{25}{9}\right]$$

7. D p. 51

<u>By definition</u>, $g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$= \lim_{h\to 0} \frac{4xh+2h^2}{h}$ (Since $g(x+h) - g(x) = 4xh + 2h^2$)

$= \lim_{h\to 0} (4x+2h) = 4x$

8. B p. 51

$f(x) = x^5 - 5x^4 + 3$

$f'(x) = 5x^4 - 20x^3$

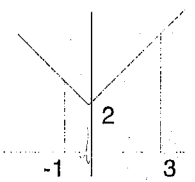
$f''(x) = 20x^3 - 60x^2 = 20x^2(x-3) \quad \Rightarrow \quad f''(x) > 0$ if and only if $x > 3$.

9. ~~C~~ D p. 52

Average Value $= \frac{1}{3+1} \int_{-1}^{3} (2 + |x|)\, dx$

$= \frac{1}{4}\left[\int_{-1}^{0} (2-x)\, dx + \int_{0}^{3} (2+x)\, dx \right] = \frac{1}{4}\left[\frac{5}{2} + \frac{7}{2}\right] = \frac{3}{2}$

$\frac{1}{4} \cdot \frac{12}{2} = \frac{12}{8}$



10. E p. 52

$v(t) = \frac{1}{1+t}$ $\qquad s(t) = \int v(t)\, dt = \ln|1+t| + C$

$s(0) = 5 \quad \Rightarrow \quad C = 5$

$s(t) = \ln|1+t| + 5 \quad \Rightarrow \quad s(3) = \ln(4) + 5$

11. A p. 52

Solution I. To find the inverse of the function $y = g(x) = \sqrt[3]{x-1}$, interchange x and y and solve for y.

$x = \sqrt[3]{y-1} \quad \Rightarrow \quad x^3 = y - 1 \quad \Rightarrow \quad y = x^3 + 1.$

Thus $f(x) = g^{-1}(x) = x^3 + 1$.

Then $f'(x) = 3x^2$.

Solution II. Since f is the inverse of g, we have $f(g(x)) = x$.

Differentiating gives: $f'(g(x)) \cdot g'(x) = 1$

Then $f'(g(x)) = \frac{1}{g'(x)}$.

Since $g(x) = (x-1)^{1/3}$, we know $g'(x) = \frac{1}{3}(x-1)^{-2/3}$.

Hence $f'(g(x)) = 3(x-1)^{2/3}$.

Since $g(x) = (x-1)^{1/3}$, this is: $f'((x-1)^{1/3}) = 3(x-1)^{2/3}$.

If we now substitute u for $(x-1)^{1/3}$, this is $f'(u) = 3u^2$.

12. A p. 53

Define the function G by $G(x) = \int_0^x \sqrt{1+t^3}\, dt$.

Then by the Second Fundamental Theorem, $G'(x) = \sqrt{1+x^3}$.
Note that $F(x) = G(\cos x)$, so we use the Chain Rule to determine $F'(x)$.
$F'(x) = G'(\cos x) \cdot [-\sin x]$
Then $F'(\frac{\pi}{2}) = G'(\cos\frac{\pi}{2}) \cdot (-\sin\frac{\pi}{2}) = G'(0) \cdot (-1) = -1$.

13. C p. 53

The slope of $y = 3x + 2$ is $m = 3$. Find the first quadrant point on the curve $y = x^3 + k$ at which the slope is 3.

$y' = 3x^2 \Rightarrow 3x^2 = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

Since we need a first quadrant point, $x = 1$, and the point on the line is $P(1,5)$. Then $y = x^3 + k$ must pass through $(1,5)$, so $k = 4$.

14. D p. 53

I.	A solution containing (0,2) never gets below the y-value of 1.	False
II.	From both above and below, as $y \to 1$, $\frac{dy}{dx} \to 0$.	True
III.	At a given value of y, $\frac{dy}{dx}$ is constant.	True

15. B p. 54

$$\frac{d}{dx}[\text{Arctan}(3x)] = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$$

16. E p. 54

$$\lim_{x\to 1} \frac{x^2+2x-3}{x^2-1} = \lim_{x\to 1} \frac{(x+3)(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \frac{x+3}{x+1} = \frac{4}{2} = 2$$

17. C p. 54

g is an antiderivative of f. By the Fundamental Theorem,

$\int_a^b g'(x)\, dx = g(b) - g(a)$. Thus, $\int_2^3 f(x)\, dx = \int_2^3 g'(x) = g(3) - g(2)$.

18. C p. 55

$y = 2e^{\cos x}$

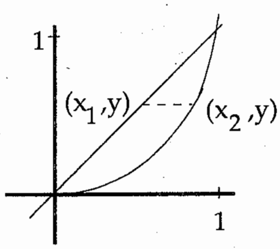
$\frac{dy}{dt} = 2e^{\cos x}(-\sin x)\frac{dx}{dt}$

When $x = \frac{\pi}{2}$ and $\frac{dy}{dt} = 5$, then $5 = 2e^0(-1)\frac{dx}{dt}$. Hence $\frac{dx}{dt} = -\frac{5}{2}$.

19. A p. 55

$$\int_1^2 \frac{dx}{x^3} = -\frac{1}{2x^2}\Big]_1^2 = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$$

20. D p. 55



The horizontal distance is the difference between the x-coordinates at a particular y.

Thus $D = x_2 - x_1 = \sqrt{y} - y$.

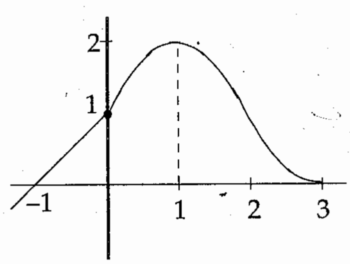
To maximize this distance function, differentiate and set equal to 0.

$\frac{dD}{dy} = \frac{1}{2\sqrt{y}} - 1 = 0 \quad \Rightarrow \quad 2\sqrt{y} = 1$

Thus the critical number is $y = \frac{1}{4}$.

The distance, for $y = \frac{1}{4}$, is $D = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$.

21. D p. 56



$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx.$$

$\int_{-1}^{0} f(x)\,dx = \frac{1}{2}$ (The area of the triangle)

$$\int_0^1 f(x)\,dx = \int_0^1 (1 + \sin \pi x)\,dx = x - \frac{1}{\pi}\cos \pi x\Big]_0^1$$

$$= \left[1 + \frac{1}{\pi}\right] - \left[0 - \frac{1}{\pi}\right] = 1 + \frac{2}{\pi}$$

The total of these two integrals is $\frac{3}{2} + \frac{2}{\pi}$

22. C p. 56

We use implicit differentiation to obtain $\frac{dy}{dx}$.

$$x^2 + 2xy - 3y = 3$$
$$2x + 2y + 2x\frac{dy}{dx} - 3\frac{dy}{dx} = 0$$
$$(2x-3)\frac{dy}{dx} = -(2x+2y)$$
$$\frac{dy}{dx} = -\frac{2x+2y}{2x-3}$$

We also need to find the y-coordinate that pairs with $x = 2$.

$$x = 2 \Rightarrow 4 + 4y - 3y = 3$$
$$\Rightarrow y = -1$$

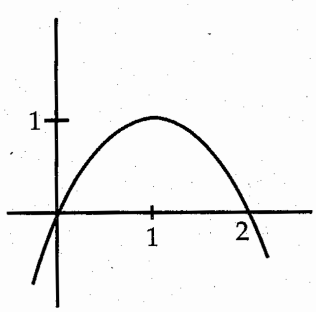
$$\Rightarrow \left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{4-2}{3-4} = -2$$

23. E p. 56

$$f(x) = x^{2/3}(5-2x)$$
$$f'(x) = \frac{2}{3}x^{-1/3}(5-2x) - 2x^{2/3}$$
$$= \frac{2}{3}x^{-1/3}[(5-2x) - 3x]$$
$$= \frac{2}{3}x^{-1/3}[5-5x]$$
$$= \frac{10}{3}x^{-1/3}(1-x)$$

This is positive when $0 < x < 1$.

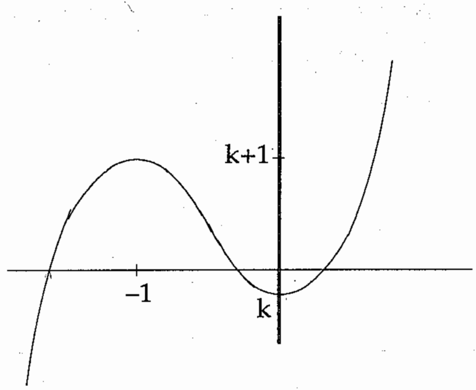
24. A p. 57



The volume of this solid formed by revolving about the x-axis is calculated using disks.

$$\pi\int_0^2 (2x-x^2)^2\,dx = \pi\int_0^2 (4x^2 - 4x^3 + x^4)\,dx$$
$$= \pi\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5}\right]_0^2$$
$$= \pi\left[\frac{32}{3} - 16 + \frac{32}{5}\right]$$
$$= \pi\frac{160-240+96}{15} = \frac{16\pi}{15}$$

25. D p. 57



$y = 2x^3 + 3x^2 + k$

The constant k only affects the vertical location of the graph of the cubic. We must adjust k so that the relative maximum and minimum points are on opposite sides of the x-axis.

$\frac{dy}{dx} = 6x^2 + 6x = 0$

The critical numbers are 0 and −1.
$y(0) = k$ and $y(-1) = 1 + k$.
We must have $k < 0$ and $k + 1 > 0$. Thus $-1 < k < 0$.

26. B p. 57

To apply the Trapezoid Rule with $n = 4$ to approximate $\int_1^5 f(x)\,dx$, we note that the width of each of the 4 subintervals is 1.
Then $T_4 = \frac{1}{2}\,[\,f(1) + 2 \cdot f(2) + 2 \cdot f(3) + 2 \cdot f(4) + f(5)\,]$.
We read the function values from the graph.
This gives $T_4 = \frac{1}{2}\,[1 + 2\cdot3 + 2\cdot1 + 2\cdot2 + 3] = \frac{1}{2}[1 + 6 + 2 + 4 + 3] = 8$.

27. C p. 58

The particle is moving to the right if the first derivative is positive.
$x'(t) = 3\cos^2 t \cdot [-\sin t]$
Then $x'(t) > 0$ if $\sin t < 0$. This first happens if $\pi < t < \frac{3\pi}{2}$.

28. C p. 58

$$\begin{aligned} f''(x) &= 2(x-2)\cdot(x-7)^3 + 3(x-7)^2\cdot(x-2)^2 \\ &= (x-2)(x-7)^2\,[2(x-7) + 3(x-2)] \\ &= (x-2)(x-7)^2\,(5x-20) = 4(x-2)(x-7)^2\,(x-4) \end{aligned}$$

$f(x)$ has a point of inflection wherever $f''(x)$ changes sign. This occurs at $x = 2$ and $x = 4$, but *not* at $x = 7$.

Exam III
Section I
Part B — Calculators Permitted

1. A p. 59

$g'(x) = \cos(\sin x)$
$g''(x) = -\sin(\sin x) \cdot \cos x$

$g'(0) = \cos(\sin 0) = \cos 0 = 1$. Since $g'(0) > 0$, g is increasing at $x = 0$.
$g''(0) = -\sin(\sin 0) \cdot \cos 0 = 0$. Then g is not concave down at $x = 0$, because g' is not decreasing at $x = 0$.
g is increasing at $x = 0$, so g cannot have a relative maximum there.

The only true statement is (I).

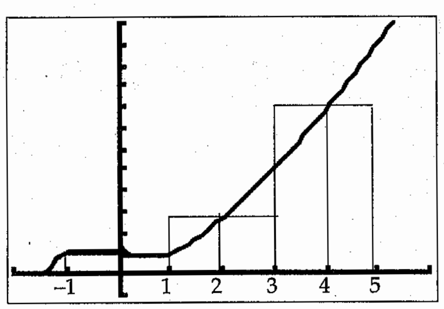
2. B p. 59

The average rate of change of a function f on the interval $[1,3]$ is $\frac{f(3)-f(1)}{3-1}$.
For this function,

$$\frac{f(3)-f(1)}{3-1} = \frac{\int_0^3 f(t)\,dt - \int_0^1 f(t)\,dt}{2}$$

$$= \frac{1}{2}\int_1^3 f(t)\,dt \approx 0.23$$

3. B p. 60

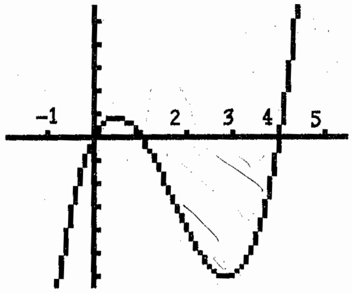


The width of each of the three rectangles is 2. Since we are forming a sum using midpoints, we evaluate the function at $x = 0$, 2, and 4.

x	0	2	4
f(x)	1	2.6458	7.8102

The midpoint approximation is:
$M_3 = 2[1 + 2.6458 + 7.8102] = 22.912$

4. D p. 60



Since the region between the graph of the curve and the x-axis consists of some area above the axis and some below, we must calculate two separate integrals.

$$\int_0^1 f(x)\,dx - \int_1^4 f(x)\,dx = 11.83$$

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5. E p. 60

Consider the function $y = \dfrac{\sin x}{x}$.

I.	It has a removable discontinuity at $x = 0$.	False
II.	$\lim\limits_{x\to\infty} \dfrac{\sin x}{x} = 0$.	True
III.	It has zeros at $x = \pm n\pi$, where n is an integer.	True

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6. C p. 61

The graph of $y = f(x+1)$ is the graph of f shifted one unit left.	IV
The graph of $y = f(x)+1$ is the graph of f shifted 1 unit up.	II
The graph of $y = f(-x)$ is the graph of f reflected in the y-axis.	III
The graph of $y = f'(x)$ is parabolic.	V

The only solution that starts IV, II, III, V is answer (C).

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7. C p. 61

Volumes of revolution about the x-axis are easily done by the disk (washer) method:

$V = \pi \int_a^b [f(x)]^2\,dx$. In this case, $f(x) = \sqrt{x}$.

$$V_{[0,4]} = \pi \int_0^4 (\sqrt{x})^2\,dx = \pi \int_0^4 x\,dx = \pi \cdot \left[\frac{x^2}{2}\right]_0^4 = 8\pi$$

$$V_{[0,k]} = \pi \int_0^k (\sqrt{x})^2\,dx = \pi \int_0^k x\,dx = \pi \cdot \left[\frac{x^2}{2}\right]_0^k = \frac{\pi k^2}{2}$$

We need $\dfrac{\pi k^2}{2} = \dfrac{1}{2}(8\pi)$. Thus $\dfrac{\pi k^2}{2} = 4\pi$, so $k^2 = 8$, and $k = 2\sqrt{2} \approx 2.83$.

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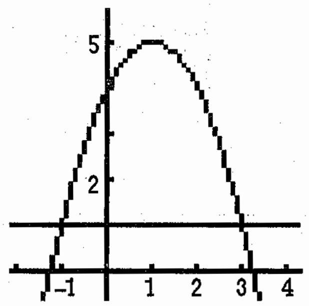
8. D p. 62

I.	f is decreasing for $-2 < x < -1$ since $f'(x) < 0$ there.	False
II.	$f'(0)$ exists, so f is continuous at $x = 0$.	True
III.	$f'(x)$ has a minimum at $x = -2$, with $f''(-2) = 0$.	True

9. C p. 62



First determine the intersection points of the two functions.

$$-x^2+2x+4 = 1$$
$$0 = x^2-2x-3$$
$$0 = (x-3)(x+1)$$
$$x = -1,\ 3$$

The area is then

$$\int_{-1}^{3} (\text{top function} - \text{bottom function})\, dx.$$

$$\int_{-1}^{3} ((-x^2+2x+4)-1)\, dx = \int_{-1}^{3} (-x^2+2x+3)\, dx \approx 10.667$$

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10. C p. 62

$f'(x) = g'(x) \quad \Rightarrow \quad f(x)-g(x) = C$

$f(1) = 2$ and $g(1) = 3 \quad \Rightarrow \quad f(x)-g(x) = -1$

The graphs do not intersect, since the graph of f is always 1 unit below the graph of g.

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11. C p. 63

I. Ave. rate of change $= \dfrac{f(3)-f(-2)}{3-(-2)} = \dfrac{2-(-1)}{3+2} = \dfrac{3}{5}$. False

II. At the point $(2,3)$, the tangent line is horizontal. True

III. The 4-subinterval left-sum approximation to $\int_{-1}^{3} f(x)\, dx$ has common width 1 and function values $-1, 0, 2, 3$; the approximation is $1 \cdot [-1+0+2+3] = 4$. True

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12. B p. 63

The distance between the ships at time t is given by $D(t) = \sqrt{W^2(t)+S^2(t)}$. This can be more simply written $D = \sqrt{W^2+S^2}$, where it is understood that all variables are functions of time t.

With all derivatives being with respect to time, we then have

$$D' = \frac{2W\cdot W' + 2S\cdot S'}{2\sqrt{W^2+S^2}} = \frac{W\cdot W' + S\cdot S'}{\sqrt{W^2+S^2}}.$$

When $t = 1$, we read from the graphs that $W = 5$ and $S = 4$.

We can also approximate the slopes of the two curves at the points where $t = 1$.

$W'(1) \approx \frac{1}{2}$ and $S'(1) \approx 1$.

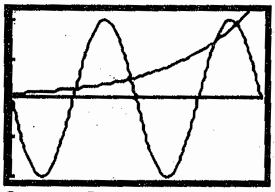
Thus, $D'(1) = \dfrac{5\cdot(1/2)+4\cdot 1}{\sqrt{5^2+4^2}} \approx 1$

13. E p. 64

$x_1'(t) = -2\sin(2t)$

$x_2'(t) = \frac{1}{2}e^{(t-3)/2}$

Graph these two velocity functions.
There are four intersection points.



$0 \le x \le 2\pi$
$-2 \le y \le 2.2$

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14. C p. 64

The line $x - 2y + 9 = 0$ has slope $m = \frac{1}{2}$. Since it is parallel to the line through $(1, f(1))$ and $(5, f(5))$, we know that $\frac{f(5) - f(1)}{5 - 1} = \frac{1}{2}$.

Since $f(1) = 2$, we then have: $\frac{f(5) - 2}{4} = \frac{1}{2}$.

Thus $f(5) - 2 = 2$, so $f(5) = 4$.

[Note: The differentiability of f, the point $(3,6)$, and the tangency of the line to the graph of f are all irrelevant.]

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15. A p. 65

Remember to use the Chain Rule.

$$\frac{d}{dx} f(x^2) = f'(x^2) \cdot 2x = 2x \cdot g(x^2)$$

$$\frac{d^2}{dx^2} f(x^2) = 2g(x^2) + 2x \cdot g'(x^2) \cdot 2x$$

$$= 2g(x^2) + 4x^2 f(3x^2)$$

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16. B p. 65

Separate variables in the differential equation.

$$\frac{dy}{dx} = 4x\sqrt{y} \quad \Rightarrow \quad \frac{dy}{\sqrt{y}} = 4x\,dx$$

$$\Rightarrow \quad 2\sqrt{y} = 2x^2 + C$$

Since the point $(1,9)$ is on the graph, we obtain

$$2\sqrt{9} = 2 \cdot 1^2 + C$$

$$6 = 2 + C$$

$$C = 4$$

Thus $2\sqrt{y} = 2x^2 + 4$, or $\sqrt{y} = x^2 + 2$.

Then when $x = 0$, we have $\sqrt{y} = 2$, so $y = 4$.

17. E p. 65

Differentiate implicitly, being careful to use the Product Rule on the right-hand side.

$$e^y = xy \quad \Rightarrow \quad e^y \cdot \frac{dy}{dx} = x \cdot \frac{dy}{dx} + y$$

$$\Rightarrow \quad (e^y - x)\frac{dy}{dx} = y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{e^y - x}$$

This result is not one of the proposed solutions, so we must do more.
Since in the original equation, $e^y = xy$, we can replace the e^y in the denominator. Then we obtain $\frac{dy}{dx} = \frac{y}{xy - x}$.

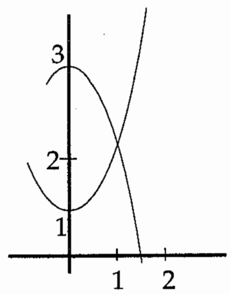
Exam III
Section II
Part A — Calculators Permitted

1. p. 67

The curves intersect in the first quadrant at x = 0.83596017. Denote that number by a.

1: Correct limits in an integral in (a), (b), or (c)

(a)



Then the area is $\int_0^a (3\cos x - e^{x^2})\,dx \approx 1.146$

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) The volume of the solid of revolution about the x-axis is done with disks:

$V_x = \pi \int_0^a \left(9\cos^2 - e^{2x^2}\right) dx$, where a is the number from part (a).

$3: \begin{cases} 2: \text{integrand and constant} \\ 1: \text{answer} \end{cases}$

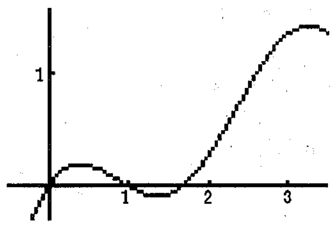
(c) Since cross sections taken perpendicular to the x-axis are squares, the cross-sectional area of the square occurring at coordinate x is $(3\cos x - e^{x^2})^2$. Then the volume of the solid described is

$V = \int_0^a (3\cos x - e^{x^2})^2\,dx$, where a again is the number found above.

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

2. p. 68

(a)



$f(x) = \ln(x+1) - \sin^2 x$ for $0 \le x \le 3$.
$y = 0 \quad \Rightarrow \quad \ln(x+1) - \sin^2 x = 0$

The function is pictured to the left. It has three zeros on $[0,3]$.

With the capabilities of the graphing calculator, the zeros are found to be at
$x = 0$
$x = 0.964$
$x = 1.684$

3: three x - intercepts

(b) Consider the graph of the derivative of f: $f'(x) = \frac{1}{x+1} - 2\sin x \cos x$. This derivative function is nonnegative valued on the intervals (0. 0.398) and (1.351, 3). Hence f is increasing there.

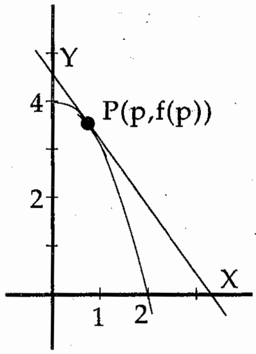
$3: \begin{cases} 1: f'(x) \ge 0 \\ 2: \text{answer} \end{cases}$

(c) The absolute maximum and absolute minimum values occur at either a critical point or an endpoint. The first derivative $f'(x) = \frac{1}{x+1} - 2\sin x \cos x$ has zeros at $x = 0.398$ and $x = 1.351$. At these critical points we have $f(0.398) = 0.184$ and $f(1.351) = -0.098$. At the endpoints we have $f(0) = 0$ and $f(3) = 1.366$. Therefore, on the interval [0, 3], the function's absolute minimum value is -0.098 and its absolute maximum value is 1.366.

$3: \begin{cases} 1: \text{identifies critical numbers and endpts as candidates} \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

3. p. 69

(a)



$f(x) = 4 - x^2 \quad \Rightarrow \quad f'(x) = -2x$

At the point $P(p, 4 - p^2)$, the tangent line has a slope of $m = -2p$. Hence an equation of the tangent line is:

$$y - (4 - p^2) = -2p(x - p).$$

This simplifies to $y = -2px + p^2 + 4$.

The y-intercept (point Y) is $(0, p^2 + 4)$.

The x-intercept (point X) is $\left(\frac{p^2+4}{2p}, 0\right)$.

The area of the triangle is $A(p) = \frac{1}{2}(p^2+4)\left[\frac{p^2+4}{2p}\right]$

Then $A(2) = \frac{1}{2}(8)(2) = 8.$

4: 1: x – intercept
1: y – intercept
1: expression for area of triangle
1: answer

(b) $A(p) = \dfrac{(p^2+4)^2}{4p}.$

Then $A'(p) = \dfrac{4p \cdot 2(p^2+4) \cdot 2p - (p^2+4)^2 \cdot 4}{16p^2} = \dfrac{4p^4 + 16p^2 - p^4 - 8p^2 - 16}{4p^2}$

$$= \frac{3p^4 + 8p^2 - 16}{4p^2}$$

$$= \frac{(3p^2-4)(p^2+4)}{4p^2}$$

The critical numbers of the function A are at $p = \pm\frac{2}{\sqrt{3}}$.

The only critical number in the interval (0,2) is $p = \frac{2}{\sqrt{3}} \approx 1.155.$

$A'(1) < 0$ and $A'(2) > 0$, therefore by the First Derivative test, $A(1.155)$ is a local minimum, and since there are no other critical points in the interval (0,2), $A(1.155)$ is the absolute minimum. The domain of A is (0, 2]. Since $\lim_{p \to 0} A(p) = \infty$, there is no maximum and $p = 1.155$ is the minimum.

5: 2: $A'(p)$
2: candidates for minimum
1: answer

Exam III
Section II
Part B — No Calculators

4. p. 70

(a) $f(x) = x^3 + px^2 + qx \quad \Rightarrow \quad f'(x) = 3x^2 + 2px + q \quad \Rightarrow \quad f''(x) = 6x + 2p$

$$\left\{\begin{array}{l} f(-1) = -8 \\ f'(-1) = 12 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} -8 = -1 + p - q \\ 12 = 3 - 2p + q \end{array}\right.$$

When we add these two equations, we obtain $4 = 2 - p$.
Thus $p = -2$. Substituting this value into one of the equations, we find that $q = 5$.

$3:\begin{cases} 1: f'(x) \\ 1: f(-1) \text{ and } f'(-1) \\ 1: \text{answer} \end{cases}$

(b) If the graph of f is to have a change in concavity at $x = 2$, then $f''(2) = 0$ and $f''(x)$ changes its sign at $x = 2$.

$f''(2) = 12 + 2p = 0 \quad \Rightarrow \quad p = -6$

Then $f''(x) = 6x - 12 = 6(x - 2)$. This <u>does</u> have a sign change at $x = 2$.

$3:\begin{cases} 1: f''(x) \\ 1: f''(2) \\ 1: \text{answer} \end{cases}$

(c) For f to be increasing everywhere, we must have $f'(x) > 0$ for all x.
Then $3x^2 + 2px + q > 0$ for all x.
$f'(x)$ is a quadratic function, opening upward.
$f'(x)$ will be positive valued $\Leftrightarrow$ f has no zeros
$\Leftrightarrow$ the discriminant of f is less than 0
$\Leftrightarrow$ $4p^2 - 12q < 0$
$\Leftrightarrow$ $p^2 < 3q$.

$3:\begin{cases} 1: f'(x) > 0 \\ 2: \text{answer} \end{cases}$

5. p. 71

(a) $s(6) = \int_0^6 v(t)\,dt = \int_0^4 v(t)\,dt + \int_4^6 v(t)\,dt = 5 - 2 = 3$

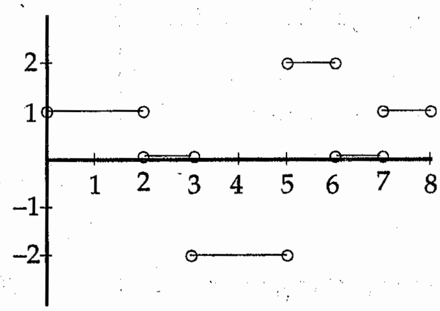
This evaluation is obtained by counting areas.

$3: \begin{cases} 1: \int_0^4 v(t)\,dt \\ 1: \int_4^6 v(t)\,dt \\ 1: \text{answer} \end{cases}$

(b) At $t = 4$, the velocity changes from being positive to being negative. Hence the car's distance from A changes from being in an increasing state to decreasing. That is, the car's direction changes.

$3: \begin{cases} 1: \text{answer} \\ 2: \text{justification} \end{cases}$

(c)



Since the acceleration is the derivative of the velocity, we need to plot a function whose values are the <u>slopes</u> of the given velocity function.

3 : graph

6. p. 72

(a) $y^3 - 3xy = 2$

Differentiating, we obtain $3y^2\frac{dy}{dx} - 3y - 3x\frac{dy}{dx} = 0.$

$$\frac{dy}{dx}(3y^2 - 3x) = 3y$$

$$\frac{dy}{dx} = \frac{3y}{3y^2 - 3x} = \frac{y}{y^2 - x}$$

$2: \begin{cases} 1: \text{implicit diff} \\ 1: \text{solves for } \frac{dy}{dx} \end{cases}$

(b) At the point (1,2), $\frac{dy}{dx}$ has the value $\frac{2}{4-1} = \frac{2}{3}$.

Therefore the tangent line has the equation $y - 2 = \frac{2}{3}(x - 1)$.

$y(1.3) = 2 + \frac{2}{3}(1.3 - 1) = 2.2$

$2: \begin{cases} 1: \text{tangent equation} \\ 1: \text{Use equation to approximate } y \text{ at } x = 1.3 \end{cases}$

(c) $$\frac{d^2y}{dx^2} = \frac{(y^2 - x)\cdot\frac{dy}{dx} - y\left(2y\frac{dy}{dx} - 1\right)}{(y^2 - x)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{(4-1)\cdot\frac{2}{3} - 8\cdot\frac{2}{3} + 2}{9} = \frac{4 - \frac{16}{3}}{9} = -\frac{4}{27}$$

$3: \begin{cases} 2: \frac{d^2y}{dx^2} \\ 1: \text{answer} \end{cases}$

(d) Since $\frac{d^2y}{dx^2} < 0$ at the point (1, 2), the graph is concave down and the tangent line lies above the curve. Hence, the point (1.3, 2.2) is an overestimate.

$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$